

EM Decay of $X(3872)$ as the $1^1D_2(2^{-+})$ charmonium

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The recently BaBar results raise the possibility that $X(3872)$ has negative parity. This makes people reconsider assigning $X(3872)$ to the $1^1D_2(c\bar{c})$ state. In this paper we give a general form of the wave function of 2^{-+} mesons. By solving the instantaneous Bethe-Salpeter equation, we get the mass spectrum and corresponding wave functions. We calculate electromagnetic decay widths of the first 2^{-+} state which we assume to be the $X(3872)$ particle. The results are $\Gamma(2^{-+}(3872) \rightarrow J/\psi\gamma) = 1.59^{+0.53}_{-0.42}$ keV, $\Gamma(2^{-+}(3872) \rightarrow \psi(2S)\gamma) = 2.87^{+1.46}_{-0.97}$ eV and $\Gamma(2^{-+}(3872) \rightarrow \psi(3770)\gamma) = 0.135^{+0.066}_{-0.047}$ keV. The ratio of branch fractions of the second and first channel is about 0.002, which is inconsistent with the experimental value 3.4 ± 1.4 . So $X(3872)$ is unlikely to be a 2^{-+} charmonium state. In addition, we obtain a relatively large decay width for $2^{-+}(3872) \rightarrow h_c\gamma$ channel which is 392^{+62}_{-111} keV.

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I. INTRODUCTION

In recent ten years B-factories have found many new heavy resonances which cannot be temporarily assigned to any charmonium or bottomonium states predicted by quark potential models. These particles are named as X , Y , Z particles. Among them $X(3872)$ is the most famous one and the one which has been studied most carefully by experiment and theory. It is also the only one that has been detected through many decay channels. Since $X(3872)$ was found by the Belle Collaboration [1], other groups like the CDF [2], DØ [3] and BaBar [4] all confirmed its existence. The following experiments put some effort into studying its quantum numbers. By detecting the EM decay channel $X \rightarrow J/\psi\gamma$, the Belle group [5] fixed the charge parity of this particle to be positive. But the parity is disputable. The Belle's results [6] favor positive parity, while the CDF [7] concludes that both positive and negative parity are possible. Recently, by analyzing the channel $X \rightarrow J/\psi\omega \rightarrow J/\psi\pi^+\pi^-\pi^0$ with the entire BaBar data sample collected at the $\Upsilon(4S)$ resonance, the BaBar Collaboration [8] concludes the negative parity for X meson is preferred. But just as Ref. [9] pointed out, positive parity assignment cannot be ruled out completely by the analysis.

For the mass of $X(3872)$ happens to lie around the $D\bar{D}^*$ threshold, many authors believe the molecule state is found. Most of the work about this particle is built on the molecule assumption or the extension of this model to mix some $c\bar{c}$ or other components [10–16]. Other fashionable models include hybrid state [17], tetraquark state [18, 19], virtual state [20, 21], etc. Although it's not as exciting as the former models, the traditional charmonium interpretation still needs careful investigation. This model faces two main problems. First, quark potential models have not predicted any state which has mass near 3872 MeV. Second, the detected strong decay channels show there is large isospin violation. But these cannot exclude this assignment completely. As pointed out by Ref. [22], large isospin violation can be explained by the kinematic suppression effects, and the

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mass of a charmonium may be affected if one considers coupled channel effects.

Before the BaBar Collaboration gave their results [8], most charmonium models assigned $\chi_{c1}(2P)$ to $X(3872)$. Now negative parity is favored by the latest experimental results, so 1D_2 state has to be reconsidered [23]. This state is the only ground state of spin-singlet D wave charmonia which have not yet been found. So finding this particle and determining its properties will be helpful to get information about interactions of quarks inside mesons. In Ref. [24], the authors have studied the 2^{-+} production in semileptonic B decays. Because EM decay channels are clean, they are good channels to find new particles. It is important to calculate EM decays of 2^{-+} in order to fix down whether or not $X(3872)$ is this state. Ref. [25] used light-front model and Ref. [26] used pNRQCD to do the radiative transition calculations. Both of them got a negative result. Ref. [27] studied the radiative transitions and the $\pi^0 D^0 \bar{D}^0$ decay mode. Ref. [28] calculated the $2^{-+}(c\bar{c})$ production cross section at the CDF using fragmentation functions. All of them got results contradict the experimental value of $X(3872)$ production.

Beside the charmonium interpretation, Ref. [29] considered the 2^- tetraquark model. Their conclusion is $X(3872)$ cannot have a 2^- tetraquark structure. Using heavy hadron chiral perturbation theory, Ref. [30] discussed the molecule interpretation of $X(3872)$ both in the 1^{++} and 2^{-+} cases. But just as [22] pointed out that even in the 1^{++} case this one pion exchange bound state is dubious, so the molecule state with larger angular momentum will be strongly disfavored.

In our previous paper [31], we have investigated the case which $X(3872)$ is assumed to be the $\chi_{c1}(2P)$ state. The EM decay ratio we get consists with the BaBar value [32]. There we also get $\text{Br}(\chi_{c1}(1P) \rightarrow J/\psi \gamma) = 35.6\%$ (this value changes to 42.4% by setting the parameters equal to new values in this paper) which is close to the experimental value $(34.4 \pm 1.5)\%$. We also studied the $J/\psi \rightarrow \eta_c \gamma$ process to make sure the method is reasonable. The branch ratio is 2.2%, while the PDG value is $(1.7 \pm 0.4)\%$. In this paper we will use the instantaneous Bethe-Salpeter (BS) [33–35] method to study the 2^{-+} scenario for $X(3872)$. Although other people's work disfavors this assignment, we think it's still necessary to give it a careful study with a different method. First, for the radiative channels different models got very different results. On one hand the discrepancy comes from the differences of these models; on the other hand people usually concentrate on determining the quantum number of this particle, thus rough estimations tend to be used. But in the radiative transition processes, the gauge invariance which is important may be violated to a large extent by using some approximations. Therefore, a careful study is needed. Second, even $X(3872)$ is not $1^1D_2(c\bar{c})$, our study will provide some useful information for this undiscovered charmonium. This will be even helpful if the 2^{--} state has similar mass with that of 2^{-+} which will bring more challenge to the experimental detection. Quark potential models have predicted that the mass range of $1^1D_2(c\bar{c})$ state is $3760 \sim 3840$ MeV [26] which is below the $D\bar{D}^*$ threshold. This will lead the particle to have a narrow decay width. Here we will not consider the threshold effects but adjust the parameters to fix the mass of ground state of 2^{-+} around 3872 MeV.

This paper is organized as follows. In the second part we give the general form of the wave function of 2^{-+} states. Then we present the instantaneous BS equation which satisfied by the 2^{-+} wave function. By solving the coupled equations we get the eigenvalues and corresponding wave functions. In the third section with Mandelstam formalism we calculate electromagnetic decay widths of this state by assuming it has the same mass with $X(3872)$. In the fourth section we give our discussions and conclusions. Appendix shows the positive wave functions and details of form factors.

II. INSTANTANEOUS BS EQUATION OF THE WAVE FUNCTION OF 2^{-+} STATE

The wave function of 2^{-+} states with mass M , momentum P and polarization tensor $\epsilon_{\mu\nu}$ has the general form

$$\phi_{2^{-+}}(q_\perp) = \epsilon_{\mu\nu} q_\perp^\mu q_\perp^\nu [f_1(q_\perp) + \frac{P}{M} f_2(q_\perp) + \frac{q_\perp}{M} f_3(q_\perp) + \frac{Pq_\perp}{M^2} f_4(q_\perp)] \gamma^5, \quad (1)$$

which satisfies constraint conditions of the Salpeter equation [34], and then we obtain the following relations

$$\begin{aligned} f_3(q_\perp) &= \frac{f_1(q_\perp) M(m_1\omega_2 - m_2\omega_1)}{q_\perp^2(\omega_1 + \omega_2)}, \\ f_4(q_\perp) &= \frac{-f_2(q_\perp) M(\omega_1 + \omega_2)}{(m_1\omega_2 + \omega_1 m_2)}, \end{aligned} \quad (2)$$

where q is the relative momentum between constituent quark and antiquark which have masses m_1 and m_2 , respectively. q_\perp is defined as $q - \frac{P}{M} P$ and ω_i has the form $\sqrt{m_i^2 - q_\perp^2}$. $f_1 \sim f_4$ are functions of $|\vec{q}|$. One can see when $m_1 = m_2$, the particle has definite C-parity. The term with f_3 which has negative C-parity disappears with the equal mass condition. So the wave function just has two independent quantities, f_1 and f_2 .

With the same method used in Ref. [36] we can get the coupled instantaneous BS equations for 2^{-+} state which have the following form

$$\begin{aligned} (M - 2\omega_1)(f_1(\vec{q}) + \frac{\omega_1}{m_1} f_2(\vec{q})) \\ = - \int d^3 \vec{k} \frac{3}{2\vec{q}^4 m_1 \omega_1} [(\vec{q} \cdot \vec{k})^2 - \frac{1}{3} \vec{q}^2 \vec{k}^2] [m_1(V_s - V_v)(m_1 f_2(\vec{k}) + \omega_1 f_1(\vec{k})) - (V_s + V_v) \vec{k} \cdot \vec{q} f_2(\vec{k})], \\ (M + 2\omega_1)(f_1(\vec{q}) - \frac{\omega_1}{m_1} f_2(\vec{q})) \\ = - \int d^3 \vec{k} \frac{3}{2\vec{q}^4 m_1 \omega_1} [(\vec{q} \cdot \vec{k})^2 - \frac{1}{3} \vec{q}^2 \vec{k}^2] [m_1(V_s - V_v)(m_1 f_2(\vec{k}) - \omega_1 f_1(\vec{k})) - (V_s + V_v) \vec{k} \cdot \vec{q} f_2(\vec{k})]. \end{aligned} \quad (3)$$

To solve above equations, we have used the Cornell potential (This phenomenological potential already reflects the main feature of the interaction between quark and anti-quark. One can modify this potential by introducing additional high order terms to get better spectra, but it will be hard to solve the BS equation. Because the wave function constructed is the most general one, even by using this simple potential we can get a reasonable result) which in momentum space can be written as

$$\begin{aligned} V(\vec{q}) &= V_s(\vec{q}) + \gamma_0 \otimes \gamma^0 V_v(\vec{q}), \\ V_s(\vec{q}) &= -(\frac{\lambda}{\alpha} + V_0) \delta^3(\vec{q}) + \frac{\lambda}{\pi^2} \frac{1}{(\vec{q}^2 + \alpha^2)^2}, \\ V_v(\vec{q}) &= -\frac{2}{3\pi^2} \frac{\alpha_s(\vec{q})}{\vec{q}^2 + \alpha^2}, \\ \alpha_s(\vec{q}) &= \frac{12\pi}{27} \frac{1}{\ln(a + \frac{\vec{q}^2}{\Lambda_{QCD}^2})}. \end{aligned} \quad (4)$$

Here we adopt the following values for the parameters [37], $a = e = 2.7183$, $\alpha = 0.06$ GeV, $\lambda = 0.21$ GeV 2 , $m_c = 1.62$ GeV, $\Lambda_{QCD} = 0.27$ GeV. One notices that we used different values of these parameters as that in [31, 36, 38], for these new values can adopt to more mesons with different quantum numbers [37], even though the former can lead to better spectra. m_c is the mass for the constituent charm quark, which is a little larger than half of the mass of η_c or J/ψ caused by the small binding energy.

Because we want to study electromagnetic decay of $2^{-+}(1D)$ as $X(3872)$, we adjust V_0 to make the first state with this quantum number to have mass equal to 3872 MeV (Since we do not consider higher order interactions, to get the mass splitting between mesons with different quantum number we introduce V_0 to fit data. Changing other parameters will affect the spectrum a lot, while changing V_0 just cause a translation of the spectrum). By doing so, we get $V_0 = -0.044$ GeV.

The normalization condition for the BS wave function is

$$\int \frac{d\vec{q}}{(2\pi)^3} \text{Tr}[\bar{\Phi}^{++} \frac{\not{P}}{M} \varphi^{++} \frac{\not{P}}{M} - \bar{\Phi}^{--} \frac{\not{P}}{M} \varphi^{--} \frac{\not{P}}{M}] = 2M, \quad (5)$$

where φ^{++} and φ^{--} are positive and negative energy part of the wave function, respectively. Their expressions can be found in the Appendix. Putting wave functions of 2^{-+} into above equation, we get its normalization condition:

$$\int \frac{d^3\vec{q}}{(2\pi)^3} \frac{8}{15} f_1 f_2 \frac{\omega_1}{M m_1} \vec{q}^4 = 1. \quad (6)$$

To solve Eq. (3), we first discretize the relative momentum \vec{q} and \vec{k} . For the wave function will approach 0 when $|\vec{q}|$ becomes large, we cut off $|\vec{q}|$ at 7.71 GeV (where the wave function is small enough). By solving the eigenvalue equation we get the mass spectrum and corresponding wave functions (in our results there is no degeneracy). Masses of the leading four states have been listed in Table I.

III. EM DECAY OF $X(3872)$ AS THE $1^1D_2(c\bar{c})$ STATE

The wave function of the 1^- state has the form [38]:

$$\begin{aligned} \varphi_{1-}(q_\perp) = & (q_\perp \cdot \epsilon)[f_1(q_\perp) + \frac{\not{P}}{M} f_2(q_\perp) + \frac{\not{q}_\perp}{M} f_3(q_\perp) + \frac{\not{P}\not{q}_\perp}{M^2} f_4(q_\perp)] + M \not{\epsilon} [f_5(q_\perp) + \frac{\not{P}}{M} f_6(q_\perp)] \\ & + (\not{q}_\perp \not{\epsilon} - q_\perp \cdot \epsilon) f_7(q_\perp) + \frac{1}{M} [\not{P} \not{\epsilon} \not{q}_\perp - \not{P} (q_\perp \cdot \epsilon)] f_8(q_\perp), \end{aligned} \quad (7)$$

where ϵ is the polarization vector of the meson, and f_i s are scalar functions of \vec{q} . When $m_1=m_2$, f_2 and f_7 equal to 0, which makes the wave function have negative C-parity.

The electromagnetic transition (see Fig. 1) amplitude is

$$T = \langle P_f \epsilon_2, k \epsilon | S | P \epsilon_1 \rangle = \frac{(2\pi)^4 e e_q}{\sqrt{2^3 \omega_\gamma E E_f}} \delta^4(P_f + k - P) \epsilon^\xi \mathcal{M}_\xi, \quad (8)$$

where $e_q = \frac{2}{3}$ is the charge of the charm quark in units of e ; ϵ , ϵ_1 and ϵ_2 are the polarization vectors (tensor) of the photon, the initial meson and the final meson, respectively; \mathcal{M}_ξ is the hadronic transition matrix element. With the method which has been proved to be gauge invariant in Ref. [39], at leading order \mathcal{M}^ξ can be written as (Here we only keep the terms contain positive energy wave functions. Other terms only contribute less than 1%).

$$\mathcal{M}^\xi = \int \frac{d\vec{q}}{(2\pi)^3} \text{Tr}[\frac{\not{P}}{M} \bar{\Phi}'^{++}(q_\perp + \alpha_2 P_{f\perp}) \gamma^\xi \varphi^{++}(q_\perp) - \bar{\Phi}'^{++}(q_\perp - \alpha_1 P_{f\perp}) \frac{\not{P}}{M} \varphi^{++}(q_\perp) \gamma^\xi], \quad (9)$$

where $P_{f\perp} = P_f - \frac{P \cdot P_f}{M^2} P$, φ^{++} and φ'^{++} are the positive energy wave functions of initial and final particle, respectively. $\bar{\Phi}'^{++}$ is defined as $\gamma^0 (\varphi^{++})^\dagger \gamma^0$. In the charmonium case, α_1 and α_2 equal to $\frac{1}{2}$.

After doing the trace and integrating out \vec{q} , we get the following form of the amplitude

$$\mathcal{M}^{\xi} = \epsilon_{\alpha\beta}\epsilon_{\mu}P_{\sigma}P_{f\delta}(\epsilon^{\xi\mu\sigma\delta}P_f^{\alpha}P_f^{\beta}t_1 + \epsilon^{\xi\sigma\delta\alpha}P^{\mu}P_f^{\beta}t_2 + \epsilon^{\xi\sigma\delta\beta}M^2g^{\alpha\mu}t_3). \quad (10)$$

One can see that there are three form factors $t_1 \sim t_3$ which are integrals of \vec{q} . Their explicit expression can be found in the Appendix. We give the results of these form factors for different states in Table II. Eq. (10) has a different form with that in Ref. [25], but one can check they are actually equivalent.

Another possible EM decay channel is $2^{-+} \rightarrow 1^{+-}\gamma$. Similar to the above calculation, we first present the wave function of 1^{+-} [40]

$$\phi_{1^{+-}}(q_{\perp}) = (q_{\perp} \cdot \epsilon)[f_1(q_{\perp}) + \frac{P}{M}f_2(q_{\perp}) + \frac{q_{\perp}}{M}f_3(q_{\perp}) + \frac{Pq_{\perp}}{M^2}f_4(q_{\perp})]\gamma_5. \quad (11)$$

One notices that it has the same Lorentz structure inside square brackets with the wave function of 2^{-+} . From Eq. (9) we get the expression for the amplitude of this transition

$$\mathcal{M}^{\xi} = \epsilon_{\mu\nu}\epsilon^{\mu}M^2(P_f^{\xi}P_f^{\nu}s_1 + P_f^{\xi}P_f^{\nu}s_2) + \epsilon_{\mu}^{\xi}\epsilon^{\mu}M^4s_3 + \epsilon_{\mu\nu}\epsilon^{\xi}M^2P_f^{\mu}P_f^{\nu}s_4 + \epsilon_{\mu\nu}\epsilon_{\delta}(P_f^{\mu}P_f^{\nu}P_f^{\xi}P_f^{\delta}s_5 + P_f^{\xi}P_f^{\delta}P_f^{\mu}P_f^{\nu}s_6) + \epsilon_{\mu}^{\xi}\epsilon_{\delta}M^2P_f^{\mu}P_f^{\delta}s_7. \quad (12)$$

This amplitude which contains seven form factors is a little bit complex compared with Eq. (10). The procedure to calculate these form factors is similar to that of above, we will not give their explicit expressions but just list their values in Table III.

TABLE I: Masses (in unit of GeV) of ${}^1D_2(c\bar{c})$ mesons with $V_0 = -0.044$ GeV. The uncertainties are gotten by varying all parameters in Cornell potential by 5%.

$n^1D_2(c\bar{c})$	1D	2D	3D	4D
Mass	$3.872^{+0.179}_{-0.180}$	$4.198^{+0.189}_{-0.183}$	$4.450^{+0.196}_{-0.189}$	$4.655^{+0.201}_{-0.196}$

TABLE II: Form factors (in unit of GeV^{-2}) of the decay process $2^{-+}(3872) \rightarrow \psi(nS)\gamma$. The uncertainties are gotten by varying all parameters in Cornell potential by 5%.

form factor	t_1	t_2	t_3
$2^{-+}(3872) \rightarrow J/\psi\gamma$	$8.36^{+0.70}_{-0.62}$	$0.447^{+0.047}_{-0.040}$	$-4.37^{+0.58}_{-0.68} \times 10^{-2}$
$2^{-+}(3872) \rightarrow \psi(2S)\gamma$	$23.5^{+1.8}_{-2.0}$	$-4.47^{+1.38}_{-1.40} \times 10^{-2}$	$3.34^{+0.47}_{-0.41} \times 10^{-2}$
$2^{-+}(3872) \rightarrow \psi(3770)\gamma$	$7.53^{+1.97}_{-0.64}$	$14.2^{+1.7}_{-1.2}$	$-0.998^{+0.084}_{-0.096}$

IV. NUMERICAL RESULTS AND DISCUSSIONS

By solving corresponding BS equations we also get the wave functions of 1^{--} states and 1^{+-} states. We use the same parameters as that used when we solve the equation of 2^{-+} except V_0 . By fitting the mass spectra, we find the best-fit values of V_0 : -0.144 GeV for 1^{+-} and -0.1756 GeV for 1^{--} .

TABLE III: Form factors (GeV^{-2}) of the decay process $2^{-+}(3872) \rightarrow h_c(1P)\gamma$. The uncertainties are gotten by varying all parameters in Cornell potential 5%.

form factor	s_1	s_2	s_3	s_4	s_5	s_6	s_7
$2^{-+}(3872) \rightarrow h_c(1P)\gamma$	$-3.96^{+0.50}_{-0.59}$	$-5.20^{+0.61}_{-0.71}$	$-0.742^{+0.117}_{-0.145}$	$2.44^{+0.38}_{-0.31}$	$20.8^{+10.9}_{-19.7}$	$28.6^{+21.9}_{-23.3}$	$4.49^{+0.73}_{-0.61}$

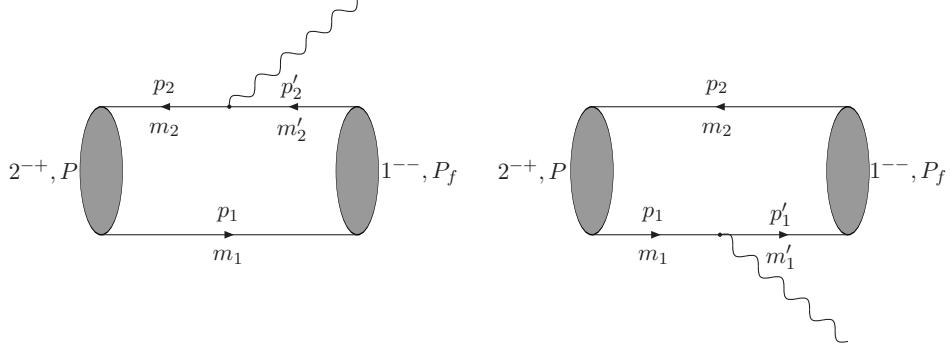


FIG. 1: Feynman diagrams of the EM decay of 2^{-+} meson.

Using Eq. (10) we can calculate decay widths of $^1D_2 \rightarrow \psi(nS)\gamma$, which have been listed in Table IV. We also calculate the uncertainties caused by varying the central values of the parameters in the Cornell potential by 5%. The form factors are presented in Table II.

Considering the upper limit of the decay width, $\Gamma_{X(3872)} < 2.3 \text{ MeV}$, we can get the theoretical predictions for the lower limit of branch fractions of these channels: $\text{Br}(X \rightarrow J/\psi\gamma) > 6.87 \times 10^{-4}$, $\text{Br}(X \rightarrow \psi(2S)\gamma) > 1.25 \times 10^{-9}$, $\text{Br}(X \rightarrow \psi(3770)\gamma) > 5.87 \times 10^{-5}$. There are experimental lower limits on the first two decay channels [41], which is much larger than our predictions. One can see that the branch fraction of X to J/ψ is one order of magnitude larger than that of X to $\psi(3770)$, while it is much larger than that of X to $\psi(2S)$. This prediction is strongly in contradiction to the experimental value [32]

$$\frac{\text{Br}(X(3872) \rightarrow \gamma\psi(2S))}{\text{Br}(X(3872) \rightarrow \gamma J/\psi)} = 3.4 \pm 1.4. \quad (13)$$

To understand our results, in Fig. 2, we plot the wave functions f_1 of 2^{-+} state, f_3 of $\psi(3770)$, f_5 of J/ψ and $\psi(2S)$ (For 2^{-+} state, f_1 almost equals to f_2 . For J/ψ and $\psi(2S)$, f_5 and f_6 which have almost equal absolute values give the main contribution, while for $\psi(3770)$ f_3 and f_4 are the large part of the wave function.). To make the wave functions dimensionless we multiply them by different quantities. One can see that the wave functions of 2^{-+} reach their maximum values when $|\vec{q}|$ takes about 0.75 GeV. Also, the wave function of $\psi(3770)$ gets its maximum value at the same $|\vec{q}|$. This state which is the third eigenstate of our 1^{--} instantaneous BS equation includes S -wave and D -wave mixing. For this state, one can see $\vec{q}^2 f_3$ has the shape of D -wave.

Actually our results above can be easily understood from the node structure of wave functions of initial and final particles. One can see there is a node in the wave function of $\psi(2S)$, which locates almost at the same position of $|\vec{q}|$ where the wave functions of 2^{-+} gets their maxima. This causes strongly decreasing of the overlap integral. While for J/ψ and $\psi(3770)$, there is no node, so we get a much larger width. We have used the same arguments in our previous work to explain the results of 1^{++}

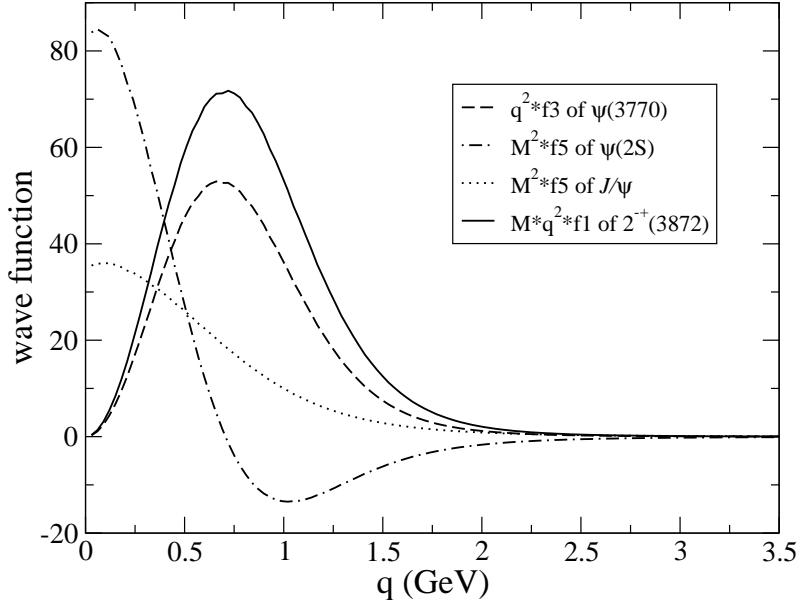


FIG. 2: Wave functions of $2^{-+}(3872)$ and of J/ψ , $\psi(2S)$, $\psi(3770)$.

decay [31]. Because the wave function of $2P$ state has a node, the cancellation happens for $1S$, not for $2S$, which leads to a result closer to experimental value.

One can see the decay width of the $J/\psi\gamma$ channel is close to that with other methods, while the $\psi(2S)\gamma$ channel has a quite different width. Ref. [25, 26, 43] consider S-D mixing by introducing a mixing angle, say $\theta = 12^\circ$ in Ref [25, 26]. Even with the same mixing angle there is still a large discrepancy which manifests the sensitivity of the results to the node structure. As for our method, all the adjustable parameters are contained in the potential and the wave function are constructed in a general way which makes it contain mixture automatically.

If we trust the results of BaBar [8], there will be no candidate for this particle in the spectrum predicted by quark potential models. The inconsistent of theoretical predictions and experimental data will force us to abandon the $1^1D_2(c\bar{c})$ assignment to $X(3872)$. But we think it's still too early to abandon the charmonium assignment. Just as Ref. [42] pointed out that experimental data interpretation is a subtle issue. We have to make sure what exactly we have measured, e.g. is there any possible that two particles with masses close to each other have been detected, such as 2^{-+} and 2^{--} ?

If $1^1D_2(c\bar{c})$ proved not to be a candidate of $X(3872)$, we would like to give predictions of this missing state. By setting $V_0 = -0.113$ GeV, we resolve the coupled equations satisfied by the 2^{-+} state. Then we get the ground state with $M = 3820$ MeV and corresponding wave functions (Quark potential models give the mass range $3760 \sim 3840$ MeV Ref. [26]. We choose 3820 MeV as an example). The decay widths of the particle with this mass are listed in Table IV. One notices that $\Gamma(1^1D_2 \rightarrow J/\psi\gamma)$ changes only a little, while for the other two channels the widths change $4 \sim 10$ times. This comes from the sensitivity to the node structure and phase space changing for the last two channels.

For the decay channel $2^{-+}(3872) \rightarrow h_c(1P)\gamma$, we also use two groups of mass values to calculate the width. For $M=3872$ MeV, the decay width is 391 keV. Within the uncertainty range, our result is consistent with those from other approaches (except

Ref. [26] where the relativistic corrections are neglected). For $M=3820\text{MeV}$, the result is 395 keV which just has a little change. This is different to that in Ref. [45], which changes more than 100 MeV when the mass of the particle changes. We can see this channel is more than two orders of magnitude wider than $J/\psi\gamma$ channel, which means this is the main EM decay channel of 1^1D_2 .

In summary, we have gotten the mass spectrum and wave functions of $1^1D_2(c\bar{c})$ states by solving the corresponding instantaneous BS equations. Then within Mandelstam formalism we calculated decay widths of different EM transition processes: $\Gamma(2^{-+}(3872) \rightarrow J/\psi\gamma) = 1.59 \text{ keV}$, $\Gamma(2^{-+}(3872) \rightarrow \psi(2S)\gamma) = 2.87 \text{ eV}$, $\Gamma(2^{-+}(3872) \rightarrow \psi(3770)\gamma) = 0.135 \text{ keV}$ and $\Gamma(2^{-+}(3872) \rightarrow h_c\gamma) = 392 \text{ keV}$. The ratio of branching fractions of the first two channels shows that $X(3872)$ is not likely to be a 2^{-+} state. More precise measurements and analyses are definitely needed for the decay widths to clarify the nature of this mysterious particle.

TABLE IV: Electromagnetic decay widths of $1^1D_2(c\bar{c})$. For our work, the results out (in) the parentheses are gotten by setting $M_X = 3872 \text{ MeV}$ (3820 MeV). Ref. [45] used $M_X = 3872 \text{ MeV}$ (out parentheses) and $M_X = 3837 \text{ MeV}$ (in parentheses). Ref. [25] and Ref. [26] did the calculation by setting $M_X = 3872 \text{ MeV}$, while Ref. [43], Ref. [44] and Ref. [46] did the calculation with $M_X = 3820, 3825$ and 3796 MeV , respectively. The uncertainties are gotten by varying all parameters in Cornell potential 5%.

Ref.	$\Gamma(1^1D_2 \rightarrow h_c\gamma) \text{ (keV)}$	$\Gamma(1^1D_2 \rightarrow J/\psi\gamma) \text{ (keV)}$	$\Gamma(1^1D_2 \rightarrow \psi(2S)\gamma) \text{ (eV)}$	$\Gamma(1^1D_2 \rightarrow \psi(3770)\gamma) \text{ (keV)}$
This work	$392_{-111}^{+62}(395)$	$1.59_{-0.42}^{+0.53}(1.08)$	$2.87_{-0.97}^{+1.46}(0.682)$	$0.135_{-0.047}^{+0.066}(0.0128)$
Ke&Li [25]		3.54	0.60	0.356
Jia <i>et al</i> [26]	$587 \sim 786$	$3.11 \sim 4.78$	$17 \sim 29$	$0.49 \sim 0.56$
S&Z [43]	288	0.699	1	
Eichten <i>et al</i> [44]	303			0.34
B&G [45]	464(344)			
Chao <i>et al</i> [46]	375			

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APPENDIX

The definition of φ^{++} can be found in Ref.[36]. From the general form of wave functions we get the positive energy part of wave functions of 2^{-+} , 1^{--} and 1^{+-} :

$$\varphi_{2^{-+}}^{++} = \epsilon_{\mu\nu} q_\perp^\mu q_\perp^\nu (A_1 \gamma_5 + A_2 \not{p} \gamma_5 + A_3 \not{q} \gamma_5), \quad (14)$$

$$\varphi_{1^{--}}^{++} = B_1 \not{\epsilon} + B_2 \not{\epsilon} \not{p} + B_3 \not{p} \not{\epsilon} \not{q}_\perp + B_4 q_\perp \cdot \epsilon + B_5 q_\perp \cdot \epsilon \not{p} + B_6 q_\perp \cdot \epsilon \not{q}_\perp + B_7 q_\perp \cdot \epsilon \not{p} \not{q}_\perp, \quad (15)$$

$$\varphi_{1^{+-}}^{++} = q_\perp \cdot \epsilon (C_1 \gamma_5 + C_2 \not{p} \gamma_5 + C_3 \not{q} \gamma_5), \quad (16)$$

where A_i s, B_i s and C_i s are scalar functions of \vec{q} which have the following forms

$$A_1 = \frac{1}{2}(f_1 + \frac{\omega_1}{m_1}f_2), \quad A_2 = \frac{m_1}{2M\omega_1}(f_1 + \frac{\omega_1}{m_1}f_2), \quad A_3 = -\frac{1}{2M\omega_1}(f_1 + \frac{\omega_1}{m_1}f_2), \quad (17)$$

$$\begin{aligned} B_1 &= \frac{M_f}{2}(f_5 - \frac{\omega_1}{m_1}f_6), \quad B_2 = -\frac{m_1}{2\omega_1}(f_5 - \frac{\omega_1}{m_1}f_6), \quad B_3 = \frac{1}{2\omega_1}(f_5 - \frac{\omega_1}{m_1}f_6), \\ B_4 &= \frac{M_f}{2m_1}(f_5 - \frac{m_1}{\omega_1}f_6) - \frac{\vec{q}^2}{2M_f m_1}(f_3 + \frac{m_1}{\omega_1}f_4), \quad B_5 = -\frac{1}{2\omega_1}(f_5 - \frac{\omega_1}{m_1}f_6), \\ B_6 &= -\frac{M_f}{2m_1\omega_1}f_6 + \frac{1}{2M_f}(f_3 + \frac{m_1}{\omega_1}f_4), \quad B_7 = -\frac{1}{2m_1\omega_1}f_5 + \frac{\omega_1}{2M_f^2 m_1}(f_3 + \frac{m_1}{\omega_1}f_4), \end{aligned} \quad (18)$$

$$C_1 = \frac{1}{2}(f_1 + \frac{\omega_1}{m_1}f_2), \quad C_2 = \frac{m_1}{2M\omega_1}(f_1 + \frac{\omega_1}{m_1}f_2), \quad C_3 = -\frac{1}{2M\omega_1}(f_1 + \frac{\omega_1}{m_1}f_2). \quad (19)$$

To get the expressions for φ^{--} , one just need to change ω_1 to $-\omega_1$ in φ^{++} .

The form factors for the $2^{-+} \rightarrow 1^{--}\gamma$ process are expressed as

$$\begin{aligned} t_1 &= (a_1 + b_1) - (a_3 + b_3) - \alpha_2 P \cdot P_f (a_8 + b_8) + \alpha_2 P \cdot P_f (a_{12} + b_{12}) - 2P \cdot P_f (a_{15} - a_{16}), \\ t_2 &= -2(a_2 + b_2) + 2\alpha_2 \frac{P \cdot P_f}{M^2} (a_5 - b_5) + 2\alpha_2 P \cdot P_f (a_{11} + b_{11}) - 2(\alpha_2 \frac{P \cdot P_f}{M})^2 (a_{14} - b_{14}), \\ t_3 &= -\frac{2}{M^2} (a_{14} + b_{14}) + 2\alpha_2 \frac{P \cdot P_f}{M^2} (a_{13} + b_{13}), \end{aligned} \quad (20)$$

where a_i s and b_i s are the integrals of \vec{q} :

$$\begin{aligned} a_1(a_8) &= \int \frac{d\vec{q}}{(2\pi)^3} A_1 B_2 (A_2 B_3) \frac{|\vec{q}|^2}{2|\vec{P}_f|^2} (3\cos^2\theta - 1), \\ a_2(a_{11}) &= -\int \frac{d\vec{q}}{(2\pi)^3} A_1 B_7 (A_3 B_7) \frac{E_f |\vec{q}|^4}{8M|\vec{P}_f|^2} (5\cos^4\theta - 6\cos^2\theta + 1), \\ a_3(a_{12}) &= \int \frac{d\vec{q}}{(2\pi)^3} A_1 B_7 (A_3 B_7) \frac{|\vec{q}|^4}{8|\vec{P}_f|^2} (5\cos^4\theta - 6\cos^2\theta + 1), \\ a_4(a_{13}) &= \int \frac{d\vec{q}}{(2\pi)^3} A_1 B_7 (A_3 B_7) \frac{|\vec{q}|^4}{8} (\cos^4\theta - 2\cos^2\theta + 1), \\ a_5(a_7, a_{10}, a_{14}) &= \int \frac{d\vec{q}}{(2\pi)^3} A_1 B_7 (A_2 B_3, A_3 B_2, A_3 B_7) \frac{|\vec{q}|^3}{2|\vec{P}_f|} (\cos^3\theta - \cos\theta), \\ a_6(a_9) &= -\int \frac{d\vec{q}}{(2\pi)^3} A_2 B_3 (A_3 B_2) \frac{E_f |\vec{q}|^3}{2M|\vec{P}_f|^3} (5\cos^3\theta - 3\cos\theta), \\ a_{15}(a_{16}) &= \int \frac{d\vec{q}}{(2\pi)^3} A_2 B_3 (A_3 B_2) \frac{|\vec{q}|^3}{2|\vec{P}_f|^3} (5\cos^3\theta - 3\cos\theta), \end{aligned} \quad (21)$$

where θ is the angle between \vec{P}_f and \vec{q} ; the arguments of A_i and B_i are \vec{q} and $\vec{q} + \alpha_2 \vec{P}_f$, respectively. To calculate b_i , one just need to change the argument of B_i from $\vec{q} + \alpha_2 \vec{P}_f$ to $\vec{q} - \alpha_1 \vec{P}_f$.

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